Appendix: "The Cyclicality of the Opportunity Cost of Employment"

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A   Model Appendix

In this Appendix we present detailed derivations underlying our analysis and show further results related to the models discussed in the main text.

A.1   Derivation of the Opportunity Cost

In this section we derive equations (10), (11), and (12) in the main text. To simplify the notation, we suppress the dependence of the value function on the capital stock K_t and the exogenous shocks $\mathbf{Z}_t$ and write the value function of the household as $W^h(e_t, \omega_t)$ instead of $W^h(e_t, \omega_t, K_t, \mathbf{Z}_t)$. In recursive form, the maximization problem of the household is:

$$W^h(e_t, \omega_t) = \max \left\{ e_t U_t^e + (1 - e_t) U_t^u - (1 - e_t)\omega_t \psi(\zeta_t) + \beta \mathbb{E}_t W^h(e_{t+1}, \omega_{t+1}) \right\}, \qquad \text{(A.1)}$$

subject to the law of motion for employment (1), the budget constraint (3), and the law of motion for the share of eligible unemployed (6). Differentiating (A.1) with respect to e_t we take:

$$\frac{\partial W^h(e_t, \omega_t)}{\partial e_t} = U_t^e - U_t^u + \omega_t \psi(\zeta_t) + \lambda_t \left[\left(\frac{1 - \tau_t^w}{1 + \tau_t^C} \right) w_t N_t - C_t^e + C_t^u - B_{n,t} - \left(\frac{1 - \tau_t^B}{1 + \tau_t^C} \right) B_{u,t} \right]$$
$$+ \beta \mathbb{E}_t \frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial e_{t+1}} \frac{\partial e_{t+1}}{\partial e_t} + \beta \mathbb{E}_t \frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial e_t}. \qquad \text{(A.2)}$$

The first derivative in equation (A.2) can be calculated using the law of motion for employment in equation (1):

$$\frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial e_{t+1}} \frac{\partial e_{t+1}}{\partial e_t} = (1 - s_t - f_t) \frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial e_{t+1}}. \qquad \text{(A.3)}$$

For the second derivative in equation (A.2), we first calculate the derivative of ω_{t+1} with respect to e_t using the law of motion for the share of eligible unemployed in equation (6):

$$\frac{\partial \omega_{t+1}}{\partial e_t} = \left(\omega_{t+1}^e - \omega_{t+1}^u \omega_t \right) \frac{s_t(1 - f_t)}{(1 - e_{t+1})^2}. \qquad \text{(A.4)}$$

Note that the household treats the job-finding probability f_t as constant when contemplating a change in the number of employed.

To calculate $\partial W^h(e_{t+1}, \omega_{t+1})/\partial \omega_{t+1}$ in equation (A.2), we first calculate the partial derivative of the value function $W^h(e_t, \omega_t)$ with respect to ω_t and then forward this equation by one period:

$$\frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial \omega_{t+1}} = (1 - e_{t+1}) \left(-\psi_{t+1} + \lambda_{t+1} \left(\frac{1 - \tau_{t+1}^B}{1 + \tau_{t+1}^C} \right) \zeta_{t+1} \tilde{B}_{t+1} \right)$$
$$+ \beta \mathbb{E}_{t+1} \frac{\partial W^h(e_{t+2}, \omega_{t+2})}{\partial \omega_{t+2}} \frac{\partial \omega_{t+2}}{\partial \omega_{t+1}}, \tag{A.5}$$

where

$$\frac{\partial \omega_{t+2}}{\partial \omega_{t+1}} = \frac{\omega_{t+2}^u (1 - f_{t+1}) u_{t+1}}{u_{t+2}}. \tag{A.6}$$

One can forward equation (A.5) to infinity to express the derivative $\partial W^h(e_{t+1}, \omega_{t+1})/\partial \omega_{t+1}$ as a function of the expected sum of discounted future net flows. To make the measurement of the opportunity cost related to UI benefits operational in the data, we impose the additional restriction that the household perceives the discounted future marginal value of increasing the current share of eligible unemployed to be constant over time.[1] Formally, we impose that household expectations in period $t + 1$ are:

$$\mathbb{E}_{t+1} \beta \frac{\partial W^h(e_{t+2}, \omega_{t+2})}{\partial \omega_{t+2}} \frac{\partial \omega_{t+2}}{\partial \omega_{t+1}} = \frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial \omega_t} \frac{\beta \lambda_{t+1}/\left(1 + \tau_{t+1}^C\right)}{\lambda_t/\left(1 + \tau_t^C\right)}. \tag{A.7}$$

Substituting (A.7) into (A.5) we obtain:

$$\frac{\partial W^h(e_{t+1}, \omega_{t+1})}{\partial \omega_{t+1}} = (1 - e_{t+1}) \left(-\psi_{t+1} + \lambda_{t+1} \left(\frac{1 - \tau_{t+1}^B}{1 + \tau_{t+1}^C} \right) \zeta_{t+1} \tilde{B}_{t+1} \right) \tilde{\Gamma}_{t+1}, \tag{A.8}$$

where $\tilde{\Gamma}_{t+1} = \left(1 - \frac{\beta \lambda_{t+1} \left(1 + \tau^C\right)}{\lambda_t \left(1 + \tau_{t+1}^C\right)} \frac{\omega_{t+1}^u (1 - f_t) u_t}{u_{t+1}} \right)^{-1}$.

The final step before deriving a recursive representation for the marginal value of employment J_t^h is to derive the utility value per recipient from UI benefits. For any t we take:

$$\lambda_t \left(\frac{1 - \tau_t^B}{1 + \tau_t^C} \right) \zeta_t \tilde{B}_t - \psi_t = \lambda_t \left(\frac{1 - \tau_t^B}{1 + \tau_t^C} \right) \zeta_t \tilde{B}_t \left(1 - \frac{\psi_t}{\psi_t' \zeta_t} \right) = \lambda_t \left(\frac{1 - \tau_t^B}{1 + \tau_t^C} \right) \zeta_t \tilde{B}_t \left(1 - \frac{1}{\alpha} \right), \tag{A.9}$$

where the first equality follows from the first-order condition (9) with respect to ζ_t and the second equality uses the definition of the elasticity of the cost function $\alpha = \psi_t' \zeta_t / \psi_t$.

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[1] Alternatively, we have used realizations of $(1 - e_{t+j}) \left(-\psi_{t+j} + \lambda_{t+j} \zeta_{t+j} \tilde{B}_{t+j} \left(1 - \tau_{t+j}^B\right) / \left(1 + \tau_{t+j}^C\right) \right)$ in the data for large j's to calculate the b_t component of the opportunity cost. This made no significant difference for our results.

Substituting equations (A.3), (A.4), (A.8), and (A.9) into equation (A.2), dividing by λ_t, and defining $J_t^h = \partial W^h\left(e_t, \omega_t, K_t, \mathbf{Z}_t\right)/\partial e_t$ leads to equations (10), (11), and (12) in the text. To measure in the data the b_t as defined in equation (11), we drop the expectations operator and use the Euler equation (8) to substitute $\frac{\beta \lambda_{t+1}\left(1+\tau_t^C\right)}{\lambda_t\left(1+\tau_{t+1}^C\right)} = \frac{1}{1+r_{t+1}}$. We measure r_{t+1} using the interest rate on 10-year U.S. Treasuries less a measure of expected inflation based on market data as constructed by the Federal Reserve Bank of Cleveland (1982-2012) and extended using survey data from the Survey of Professional Forecasters (1979-1982), the Michigan Survey of Consumers (1978-1979), and ex post inflation prior to 1978.

A.2 The MP/RBC Model

In this Appendix we first present the equations that describe the MP/RBC model. We simplify the exposition relative to the main text by assuming the b_t component of the opportunity cost is exogenous and that taxes are constant. Additionally, here we only present the version of the model with fixed utility costs associated with working. In the main text we also discussed the case in which fixed costs associated with working are denominated in units of time.

After presenting the model equations, we then show how to derive some key equations related to firms' optimization problem, the Nash bargaining solutions, and market tightness in steady state. Finally, we calibrate the model and present simulations from the calibrated model.

A.2.1 Model Equations

The model consists of 20 equations in 20 endogenous variables. The endogenous variables are m, v, e, f, q, u, θ, C^e, L^e, λ, C^u, N, R, K, I, z, Y, p, x, and w.

Labor Market Flows and Stocks.

$$m_t = M v_t^\eta (1 - e_t)^{1-\eta}. \tag{A.10}$$

$$f_t = \frac{m_t}{1 - e_t}. \tag{A.11}$$

$$q_t = \frac{m_t}{v_t}. \tag{A.12}$$

$$e_{t+1} = f_t + (1 - s - f_t)e_t. \tag{A.13}$$

$$u_t = 1 - e_t. \tag{A.14}$$

$$\theta_t = \frac{v_t}{1 - e_t}. \tag{A.15}$$

Household Optimization.

$$\frac{\partial U(C_t^e, L_t^e)}{\partial C_t^e} = \lambda_t. \tag{A.16}$$

$$\frac{\partial U(C_t^u, L^u)}{\partial C_t^u} = \lambda_t. \tag{A.17}$$

$$L_t^e + N_t = L^u. \tag{A.18}$$

$$\frac{\lambda_t}{1 + \tau^C} = \mathbb{E}_t \beta \left(\frac{\lambda_{t+1}}{1 + \tau^C} \right) (R_{t+1} + 1 - \delta). \tag{A.19}$$

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{A.20}$$

$$z_t = b_t + \frac{(U_t^u - \lambda_t C_t^u) - (U_t^e - \lambda_t C_t^e)}{\lambda_t} + \frac{\text{FC}}{\lambda_t}. \tag{A.21}$$

Firm Optimization.

$$Y_t = A_t K_t^\nu (e_t N_t)^{1-\nu}. \tag{A.22}$$

$$\nu \frac{Y_t}{K_t} = R_t. \tag{A.23}$$

$$p_t = (1 - \nu)\frac{Y_t}{e_t}. \tag{A.24}$$

$$x_t = \frac{p_t}{N_t}. \tag{A.25}$$

$$\frac{\kappa}{q_t} = \mathbb{E}_t \frac{\beta \lambda_{t+1}}{\lambda_t} \left(p_{t+1} - w_{t+1} N_{t+1} + \frac{\kappa(1 - s_{t+1})}{q_{t+1}} \right). \tag{A.26}$$

Nash Bargaining.

$$\frac{\partial U(C_t^e, L_t^e)}{\partial L_t^e} = \left(\frac{1 - \tau^w}{1 + \tau^C} \right) x_t. \tag{A.27}$$

$$w_t N_t = \mu p_t + \left(\frac{1 + \tau^C}{1 - \tau^w} \right) (1 - \mu)z_t + \mu \kappa \theta_t. \tag{A.28}$$

Resource Constraint.

$$Y_t = e_t C_t^e + (1 - e_t)C_t^u + C^o + I_t + \kappa v_t. \tag{A.29}$$

A.2.2 Firm's Optimization Problem

In this section we derive equation (38) in the main text and equation (A.26) in this Appendix. The firm's problem is to maximize the expected present value of dividend flows:

$$\max_{K_t, v_t} W^f \left(e_t, \mathbf{Z}_t \right) = F_t(K_t, e_t N_t) - R_t K_t - w_t e_t N_t - \kappa v_t + \mathbb{E}_t \tilde{\beta}_{t+1} W^f \left(e_{t+1}(v_t), \mathbf{Z}_t \right), \quad \text{(A.30)}$$

subject to the law of motion for employment $e_{t+1} = (1 - s_t)e_t + m_t = (1 - s_t)e_t + q_t v_t$. We denote by $\tilde{\beta}_t = \beta \lambda_t / \lambda_{t-1}$ the stochastic discount factor of the household. The firm treats $\tilde{\beta}_t$, the vacancy-filling probability q_t, and market tightness θ_t as given. Denote the pre-tax marginal product of employment by $p_t = \partial F_t / \partial e_t$.

The first-order condition with respect to capital is $\partial F_t / \partial K_t = R_t$. The first-order condition with respect to vacancies is:

$$- \kappa + \mathbb{E}_t \tilde{\beta}_{t+1} \frac{\partial W^f \left(e_{t+1}, \mathbf{Z}_{t+1} \right)}{\partial e_{t+1}} \frac{\partial e_{t+1}}{\partial v_t} = 0 \implies \frac{\kappa}{q_t} = \mathbb{E}_t \tilde{\beta}_{t+1} \frac{\partial W^f \left(e_{t+1}, \mathbf{Z}_{t+1} \right)}{\partial e_{t+1}}. \quad \text{(A.31)}$$

The Envelope condition is:

$$\frac{\partial W^f \left(e_t, \mathbf{Z}_t \right)}{\partial e_t} = p_t - w_t N_t + \frac{\kappa (1 - s_t)}{q_t}. \quad \text{(A.32)}$$

Forwarding the Envelope condition by one period we obtain:

$$\mathbb{E}_t \tilde{\beta}_{t+1} \frac{\partial W^f \left(e_{t+1}, \mathbf{Z}_{t+1} \right)}{\partial e_{t+1}} = \mathbb{E}_t \tilde{\beta}_{t+1} \left(p_{t+1} - w_{t+1} N_{t+1} + \frac{\kappa (1 - s_{t+1})}{q_{t+1}} \right). \quad \text{(A.33)}$$

Finally, substituting equation (A.33) into (A.31) we obtain (A.26). Defining the firm's marginal value from an additional employed worker as $J_t^f = \partial W^f \left(e_t, \mathbf{Z}_t \right) / \partial e_t$ and using equations (A.31) and (A.32) we obtain equation (38) in the main text.

A.2.3 Nash Bargaining

In this Appendix we show how to obtain equations (A.27) and (A.28) that characterize the solution to the Nash bargaining. The total surplus associated with the formation of a job-worker pair (measured in units of wealth) is:

$$S_t = \left(1 + \tau^C \right) \frac{J_t^h}{\lambda_t} + J_t^f, \quad \text{(A.34)}$$

where the marginal value of employment for the household J_t^h is given by equation (10) and the marginal value of employment for the firm J_t^f is given by equation (38) in the main text. The firm and the household bargain over hours per employed N_t and the wage w_t to maximize:

$$\max_{N_t, w_t} \left\{ \mu \log \left((1 + \tau^C) \frac{J_t^h}{\lambda_t} \right) + (1 - \mu) \log \left(J_t^f \right) \right\}. \tag{A.35}$$

The first-order conditions with respect to N_t and w_t are given by:

$$\frac{\partial \left((1 + \tau^C) \frac{J_t^h}{\lambda_t} \right)}{\partial N_t} + \left(\frac{1 - \tau^w}{1 + \tau^C} \right) \frac{\partial J_t^f}{\partial N_t} = 0. \tag{A.36}$$

$$(1 - \mu) \frac{J_t^h}{\lambda_t} = \mu \left(\frac{1 - \tau^w}{1 + \tau^C} \right) J_t^f. \tag{A.37}$$

Equation (A.36) says that hours per employed are set to maximize the joint surplus, conditional on tax distortions. To obtain equation (A.27), we use equations (10) and (38) to compute the derivatives of the marginal values with respect to N_t in equation (A.36).

Equation (A.37) is the surplus sharing rule. This condition yields the wage equation (A.27). To derive this equation, start with equation (A.37) and substitute equations (10) and (38) for the marginal values. The derivation also uses the fact that:

$$\mathbb{E}_t \beta \frac{J_{t+1}^h}{\lambda_t} = \left(\frac{\mu}{1 - \mu} \right) \left(\frac{1 - \tau^w}{1 + \tau^C} \right) \mathbb{E}_t \tilde{\beta}_{t+1} J_{t+1}^f = \left(\frac{\mu}{1 - \mu} \right) \left(\frac{1 - \tau^w}{1 + \tau^C} \right) \frac{\kappa}{q_t}. \tag{A.38}$$

A.2.4 Steady State Tightness

In this section we derive tightness θ in steady state and the elasticity $\epsilon(\theta, p)$ in equation (40) in the main text. Start from the first-order condition for vacancies (A.26) in steady state:

$$\frac{\kappa}{q} = \beta \left(p - wN + \frac{\kappa(1 - s)}{q} \right). \tag{A.39}$$

Substituting into this equation the equilibrium wage payment from equation (A.28), using $q = f/\theta$ to substitute out q, and rearranging the resulting expression we obtain steady state tightness:

$$\theta = \left(\frac{1 + \tau^C}{1 - \tau^w} \right) \left(\frac{1}{\kappa} \right) \left(\frac{\beta(1 - \mu)f}{1 - \beta(1 - s) + \beta\mu f} \right) \left[\left(\frac{1 - \tau^w}{1 + \tau^C} \right) p - z \right]. \tag{A.40}$$

Equation (40) in the main text is obtained after differentiating θ with respect to p and using the fact that $\epsilon(f, p) = \eta\epsilon(\theta, p)$.

A.2.5 Simulations of the MP/RBC Model

Preferences. Let $s \in \{e, u\}$ be a labor force indicator. We simulate the MP/RBC for the following preferences:

$$\text{CFE:} \qquad U_t^s = \frac{1}{1 - \rho} \left((C_t^s)^{1-\rho} \left(1 - (1 - \rho) \frac{\chi \epsilon}{1 + \epsilon} (N_t^s)^{1 + \frac{1}{\epsilon}} \right)^{\rho} - 1 \right),$$

$$\text{CD:} \qquad U_t^s = \frac{1 - \chi}{1 - \rho} (C_t^s)^{1-\rho} (L^u - N_t^s)^{\frac{\chi(1-\rho)}{1-\chi}},$$

$$\text{NLD:} \qquad U_t^s = \frac{1}{1 - \rho} \left((C_t^s)^{1-\rho} - 1 \right).$$

The purpose of the no labor disutility preferences ("NLD") is to nest the models of Shimer (2005) and Hagedorn and Manovskii (2008) in which the opportunity cost z is constant. Without labor disutility, we always have $z_t = b_t$, so we adjust the level of b to achieve any desired level of z. Note that all utility functions are defined without subtracting any fixed utility costs associated with working. Fixed utility costs from working enter the definition of z_t in equation (A.21).

Shocks. The model is driven by TFP shocks:

$$A_t = A^* \exp \left(u_t^A \right) \qquad \text{with} \quad u_t^A = \rho^A u_{t-1}^A + \sigma^A \epsilon_t^A. \tag{A.41}$$

We choose $\rho^A = 0.90$ and $\sigma^A = 0.007$.

Calibration of Parameters. We set externally $\beta = 0.99$, $s = 0.045$, $\mu = 0.6$, $\eta = 0.4$, $\nu = 0.333$, $\delta = 0.025$, $\tau^C = 0.096$, $\tau^w = 0.209$ and $\epsilon = 0.7$. For CFE or CD preferences, we set $b = 0.058$ and calibrate 7 parameters (M, A^*, ρ, χ, FC, κ, C^o) to hit 7 targets. The targets are steady state values of $f = 0.704$, $q = 0.71$, $p = (1 + \tau^C)/(1 - \tau^w)$, $N = 1$, $C^e = 0.681$, $C^u = 0.540$, and some target level for z. For NLD preferences, we set externally $\rho = 1.52$ and $N = 1$. The target for consumption changes to $C^e = C^u = 0.675$. Finally, we choose b to target any level of desired z. For all models, the precise mapping from targeted moments to parameters is available upon request.

Results. In Table A.1 we present results from 8 models. The first two models feature CFE preferences. In Model 1 we set fixed costs to zero, whereas in Model 2 we choose the fixed costs

Table A.1: Statistics from Simulations

Model	Level of z	$\epsilon(\hat{z}_t, \hat{p}_t)$	$\epsilon(\hat{u}_{t+1}, \hat{p}_t)$	$\text{sd}(\hat{u}_t)/\text{sd}(\hat{Y}_t)$
1. CFE and FC $= 0$	0.470	0.88	-0.34	0.33
2. CFE and FC > 0	0.955	0.75	-1.53	1.70
3. CD, $L^u = 2.64$, and FC $= 0$	0.755	0.79	-0.51	0.48
4. CD, $L^u = 2.64$, and FC > 0	0.955	0.73	-1.72	1.74
5. CD, $L^u = 4.33$, and FC $= 0$	0.838	0.82	-0.51	0.55
6. CD, $L^u = 4.33$, and FC > 0	0.955	0.78	-1.41	1.48
7. NLD, FC $= 0$, and $b = 0.400$	0.400	0.00	-0.54	0.51
8. NLD, FC $= 0$, and $b = 0.955$	0.955	0.00	-6.62	5.16

to target a steady state $z = 0.955$. Models 3 to 6 feature CD preferences. In Models 3 and 4 we set the endowment of time to $L^u = 2.64$. In Models 5 and 6 we set the endowment of time to $L^u = 4.33$. In Models 3 and 5 we set fixed costs to zero, whereas in Models 4 and 6 we choose the fixed costs to target a steady state $z = 0.955$. Finally, Models 7 and 8 feature NLD preferences. In Model 7 we set $b = 0.400$ to target the level of z used in Shimer (2005). In Model 8 we set $b = 0.955$ to target the level of z used in Hagedorn and Manovskii (2008).

Table A.1 presents summary statistics from our simulations. For a high level of the opportunity cost $z = 0.955$, unemployment responds significantly to changes in p when z is constant (row 8) as argued by Hagedorn and Manovskii (2008). However, unemployment becomes much less responsive when z comoves with p even when the level of z is high (as in rows 2, 4, and 6). Row 7 shows that with a constant but low z, unemployment is again not very responsive as argued by Shimer (2005).

A.3   The Hall and Milgrom (2008) Model

Here we repeat elements from Hall and Milgrom (2008) that we borrow for our analysis in Section 7.2. The driving force in the model is productivity p_i where i is a discrete stationary

state variable $i \in [1, 2, ..., N]$ with transition matrix $\pi_{i,i'}$. Workers and employers are risk neutral and discount future flows at a rate r.

The only additional feature that we introduce relative to Hall and Milgrom (2008) is that we allow the flow opportunity cost of employment to potentially vary across states, z_i. Unemployed's value $\tilde{U}_i^u$ is given by:

$$\tilde{U}_i^u = z_i + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} \left[f(\theta_i) \left(\tilde{w}_{i'} + \tilde{V}_{i'} \right) + (1 - f(\theta_i)) \tilde{U}_{i'}^u \right], \tag{A.42}$$

where $f(\theta_i)$ denotes the job finding probability, $\tilde{w}_{i'}$ denotes the present value of wages at the beginning of the next state i', and $\tilde{V}_{i'}$ denotes the value for the rest of the career conditional on being matched:

$$\tilde{V}_i = \frac{1}{1+r} \sum_{i'} \pi_{i,i'} \left[s \tilde{U}_{i'}^u + (1 - s) \tilde{V}_{i'} \right]. \tag{A.43}$$

The present value of output produced over the course of a job is:

$$\tilde{p}_i = p_i + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (1 - s) \tilde{p}_{i'}. \tag{A.44}$$

The zero-profit condition is given by:

$$q(\theta_i) \left(\tilde{p}_i - \tilde{w}_i \right) = \kappa. \tag{A.45}$$

Equations (A.42) to (A.45) are common in both the Nash bargaining model and in the alternating-offer bargaining model. With Nash bargaining the wage equation is given by:

$$\tilde{w}_i = \mu \tilde{p}_i + (1 - \mu) \left(\tilde{U}_i^u - \tilde{V}_i \right). \tag{A.46}$$

In the alternating-offer model, we need to simultaneously solve for the offered payment $\tilde{w}_i$ from the employer and the counteroffer from the worker $\tilde{w}_i'$. The two equations replacing (A.46) are:

$$\tilde{w}_i + \tilde{V}_i = \delta \tilde{U}_i^u + (1 - \delta) \left[z_i + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} \left(\tilde{w}_{i'}' + \tilde{V}_{i'} \right) \right], \tag{A.47}$$

$$\tilde{p}_i - \tilde{w}_i' = (1 - \delta) \left[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} \left(\tilde{p}_{i'}^e - \tilde{w}_i \right) \right], \tag{A.48}$$

where δ denotes the probability that bargaining will exogenously terminate in the next period and γ denotes a cost that the employer incurs each period that bargaining continues.

Following Hall and Milgrom (2008), we discretize the productivity process in $N = 5$ points and use the transition matrix $\pi_{i,i'}$ shown in their Table 1. The Nash bargaining model consists of 25 equations that can be solved for 25 unknowns ($\tilde{U}_i^u$, $\tilde{V}_i$, θ_i, $\tilde{p}_i$, and $\tilde{w}_i$ for $i = 1, ..., 5$) and the alternating-offer bargaining model consists of 30 equations that can be solved for 30 unknowns ($\tilde{U}_i^u$, $\tilde{V}_i$, θ_i, $\tilde{p}_i$, $\tilde{w}_i$, and $\tilde{w}_i'$ for $i = 1, ..., 5$).

To solve these systems we use the Hall and Milgrom (2008) parameters listed in their Table 6. Hall and Milgrom (2008) discuss a separation rate of $s = 0.14/100$ in the text and show a separation rate of $s = 0.10/100$ in their Table 6. We set the separation rate to $s = 0.1383/100$ to make the steady state $\tilde{p}$ close to 636, which is the equilibrium value cited in Hall and Milgrom (2008).

A.4   Directed Search and Wage Posting Model

To ease the exposition in this section, we make some simplifying assumptions. First, we abstract from capital. Second, we also abstract from the UI take-up margin and subsume the take-up costs and expiration adjustment into the variable b_t in the budget constraint. Finally, we assume that all taxes are constant.

In recursive form, the household's maximization problem is:

$$W^h(\{e_t(i)\}) = \max \left\{ \sum_{i=1}^{M} [e_t(i)U_t^e(i) + u_t(i)U_t^u(i)] + \beta\mathbb{E}_t W^h(\{e_{t+1}(i)\}) \right\}, \tag{A.49}$$

subject to the constraints:

$$(1 + \tau^C) \sum_{i=1}^{M} [e_t(i)C_t^e(i) + u_t(i)C_t^u(i)] = (1 - \tau^w) \sum_{i=1}^{M} w_t(i)N_t(i)e_t(i) + \sum_{i=1}^{M} (1 - e_t(i))b_t, \tag{A.50}$$

$$e_{t+1}(i) = (1 - s_t)e_t(i) + f_t(i)u_t(i), \tag{A.51}$$

$$\sum_{i=1}^{M} u_t(i) = 1 - \sum_{i=1}^{M} e_t(i). \tag{A.52}$$

The marginal value to the household of an additional worker in submarket i is:

$$\frac{J_t^h(i)}{\lambda_t} = \left(\frac{1 - \tau^w}{1 + \tau^C}\right) w_t(i)N_t(i) - z_t(i) + \mathbb{E}_t \frac{\beta\lambda_{t+1}}{\lambda_t}(1 - s_t - f_t(i))\frac{J_{t+1}^h(i)}{\lambda_{t+1}}. \quad \text{[Text eq. (41)]}$$

The household optimally allocates searchers across submarkets. We first write the marginal contribution to the household value function of moving a searcher from submarket i' to submarket i:

$$\frac{\partial W^h(\{e_t(i)\})}{\partial u_t(i)} - \frac{\partial W^h(\{e_t(i')\})}{\partial u_t(i')} = \beta \mathbb{E}_t \left[f_t(i) J_{t+1}^h(i) - f_t(i') J_{t+1}^h(i') \right], \tag{A.53}$$

where in deriving equation (A.53) we have used the risk-sharing condition for the unemployed which implies $C_t^u(i) = C_t^u$ and $U_t^u(i) = U_t^u$. Optimization requires setting the left-hand side of equation (A.53) to zero. Therefore, we obtain:

$$F_t^* = f_t(i) \mathbb{E}_t \tilde{\beta}_{t+1} \frac{J_{t+1}^h(i)}{\lambda_{t+1}} = f_t(i') \mathbb{E}_t \tilde{\beta}_{t+1} \frac{J_{t+1}^h(i')}{\lambda_{t+1}} \ \forall i, i', \tag{A.54}$$

where the value F_t^* is an equilibrium object that households and firms take as given.

A firm considering whether to post a vacancy seeks to maximize its value across all possible submarkets i:

$$V_t(i) = -\kappa + q(\theta_t(i)) \mathbb{E}_t \left[\tilde{\beta}_{t+1} J_{t+1}^f(i) \right], \tag{A.55}$$

where

$$J_t^f(i) = (x_t - w_t(i)) N_t(i) + (1 - s) \mathbb{E}_t \tilde{\beta}_t J_{t+1}^f(i). \tag{A.56}$$

The firm's optimization problem is subject to equation (41) in the text and equation (A.54). The firm anticipates free entry will drive the value of a vacancy to zero in the next period. Therefore, the objective function becomes:

$$V_t(i) = -\kappa + q(\theta_t(i)) \mathbb{E}_t \tilde{\beta}_{t+1} \left[(x_{t+1} - w_{t+1}(i)) N_{t+1}(i) + \frac{(1 - s_{t+1})\kappa}{q(\theta_{t+1}(i))} \right]. \tag{A.57}$$

We assume that the economy enters into period t with $e_t(i) = e_t(i') \ \forall i, i'$ and that agents anticipate symmetric tightness across submarkets in period $t+1$, $\theta_{t+1}(i) = \theta_{t+1}$. Then substituting out the wage payment from equation (41) in the text and equation (A.54) into equation (A.57) yields:

$$V_t(i) = -\kappa + q(\theta_t(i)) \mathbb{E}_t \tilde{\beta}_{t+1} \left[x_{t+1} N_{t+1}(i) + \frac{(1 - s_{t+1})\kappa}{q(\theta_{t+1})} \right] -$$
$$q(\theta_t(i)) \left(\frac{1 + \tau^C}{1 - \tau^w} \right) \left[\frac{1}{f(\theta_t(i))} F_t^* + \mathbb{E}_t \tilde{\beta}_{t+1} \left(z_{t+1}(i) - \frac{(1 - s_{t+1} - f_{t+1})}{f_{t+1}} F_{t+1}^* \right) \right]. \tag{A.58}$$

We denote by $Q_t(i) = \theta_t(i)^{-1}$ the queue length in submarket i. The first-order conditions with respect to $Q_t(i)$ and $N_{t+1}(i)$ are:

$$q'(Q_t(i))\mathbb{E}_t\tilde{\beta}_{t+1}\left[\left(\frac{1-\tau^w}{1+\tau^C}\right)\left(x_{t+1}N_{t+1}(i) + \frac{(1-s_{t+1})\kappa}{q(\theta_{t+1})}\right) - \left(z_{t+1}(i) - \frac{(1-s_{t+1}-f_{t+1})}{f_{t+1}}F_{t+1}^*\right)\right] = F_t^*, \quad \text{(A.59)}$$

$$-\mathbb{E}_t\tilde{\beta}_{t+1}\frac{1}{\lambda_{t+1}}\frac{\partial U(C_{t+1}^e(i), N_{t+1}(i))}{\partial N_{t+1}(i)} = \mathbb{E}_t\tilde{\beta}_{t+1}x_{t+1}^\tau. \quad \text{(A.60)}$$

We now show that $N_t(i) = N_t$, $C_t^e(i) = C_t^e$, $z_t(i) = z_t$, and $\theta_t(i) = \theta_t$. First, we invert the risk-sharing condition for the employed $\partial U\left(C_{t+1}^e(i), N_{t+1}(i)\right)/\partial C_{t+1}^e(i) = \lambda_{t+1}$ for all i and write $C_{t+1}^e(i) = C(\lambda_{t+1}, N_{t+1}(i))$. Substituting $C_{t+1}^e(i) = C(\lambda_{t+1}, N_{t+1}(i))$ into equation (A.60) implies that all firms post the same hours, $N_t(i) = N_t$ in all periods t. Imposing symmetric hours in the risk-sharing conditions for the employed, we obtain $C_t^e(i) = C_t^e$. Given that $C_t^e(i) = C_t^e$, $C_t^u(i) = C_t^u$, and $N_t(i) = N_t$, we obtain $z_t(i) = z_t$ in all periods t. Finally, imposing $z_{t+1}(i) = z_{t+1}$ and $N_{t+1}(i) = N_{t+1}$ to equation (A.59), we obtain that every submarket has the same $Q_t(i) = Q_t$ and the same $\theta_t(i) = \theta_t$.

Finally, we solve for the steady state of the model. Noting that $q'(Q) = \frac{\partial q(\theta)}{\partial \theta}\frac{d\theta}{d\theta^{-1}} = -q'(\theta)\theta^2 = -f(\theta)\epsilon_{q\theta}$, we use equation (41) in the text along with equation (A.54), equation (A.57), and equation (A.59) to form a system of three equations in the three unknowns θ, wN, and F^*:

$$F^* = f(\theta)\beta\left[\left(\frac{1-\tau^w}{1+\tau^C}\right)wN - z + \frac{(1-s-f(\theta))}{f(\theta)}F^*\right], \quad \text{(A.61)}$$

$$\kappa = q(\theta)\beta\left[p - wN + \frac{(1-s)\kappa}{q(\theta)}\right], \quad \text{(A.62)}$$

$$F^* = -f(\theta)\epsilon_{q\theta}\beta\left[p^\tau + \frac{(1-s)\kappa^\tau}{q(\theta)} - \left(z - \frac{(1-s-f(\theta))}{f(\theta)}F^*\right)\right]. \quad \text{(A.63)}$$

Solving this system for θ and differentiating with respect to p yields equation (42) in the text.

B   Data Appendix

This Appendix describes the data sources used in the paper, details some further issues related to the construction of the samples and the definitions of our variables, and presents the details of our estimates in the model with heterogeneity.

B.1 Data Sources

CPS March Supplement. The Census Bureau administers the March CPS Social and Economic Supplement to the approximately 57,000 households in the basic monthly CPS sample and, in recent years, to an additional 42,000 households drawn from surrounding months. An address selected for the monthly CPS sample will be asked to complete interviews in eight calendar months. The CPS employs a rotating sample, where the household will participate in the survey for four consecutive months, not participate for eight months, and then reenter the sample for four more months. Addresses that complete their first, second, third, or fourth interview in March of year t will therefore also appear in the sample in March of year $t + 1$. Hence up to half of the respondents in the March Supplement drawn from the basic monthly CPS sample appear in consecutive Supplements.

The CPS March Supplement documentation files contain instructions for matching observations in consecutive Supplements. We follow Madrian and Lefgren (1999) in validating matches using demographic characteristics reported in both years. In particular, we require that matched observations report the same sex and race in both years, report levels of educational attainment no more than one year apart and non-decreasing, and report a difference in age of not more than two years and non-decreasing. Matching of the 1995 and 1996 Supplements is not possible because of the introduction of the 1990 Census design sample in the 1996 Supplement. We start our measurement of benefits in 1989 because the CPS does not separately record the various types of benefits before then.

CPS Basic Monthly. Our sample of CPS Basic Monthly microdata files covers the period 1968-2013, with some gaps during the period 1968-75. The 1968-75 files come from ICPSR. Specifically, we include October in every year beginning in 1968, May in every year beginning in 1969, March in 1968-69 and 1971-75, and June in 1971 and 1973-75. We linearly interpolate between the missing months to obtain continuous time series.

SIPP. The SIPP began in 1979 as a longitudinal survey with the objective of interviewing individuals in a representative sample of households once every four months for a 32 month period. In 1996 the survey underwent a major redesign, including increasing the size of the initial sample, increasing the interview period to four years, and oversampling households from high poverty areas. We use the 1996, 2001, 2004, and 2008 panels. In each wave of a panel, the household reports employment status and income for each of the previous four months. We average employment status and aggregate income for each four month period and then take first differences to obtain equation (15). Because of gaps between panels, the SIPP does not have any observations in certain months of 2000, 2001, 2004, and 2008.

CE. The Consumer Expenditure Survey interviews households every three months for up to five interviews. The first interview initiates the household into the sample and collects basic demographic information. At interviews 2-5, the respondent reports expenditure over the prior three months on a detailed set of categories designed to cover the universe of household expenditure. Interview 2 and interview 5 collect information about weeks worked over the twelve month period ending at the time of the interview. Hence at the fifth interview, we have information on both weeks worked and total expenditure over the previous year.

Our CE sample covers 1983-2012 and consists of respondents where the household completed all four interviews with a household head between 30 and 55 years old at the time of the final interview. We additionally restrict the sample to households which did not change size over the 12 month interview period, the head did not work in farming, forestry, fishing, or armed services, and in which food expenditure over the year exceeds 500 dollars in 2009 dollars.

Our definition of nondurable goods and services less housing, health, and education follows conventional NIPA definitions. We use a crosswalk provided by Cooper (2010) to map the PSID categories of clothing, recreation, and vacation into CE UCC codes. We assign households to the calendar year containing the majority of their reporting period.

PSID. The PSID began in 1968. The initial sample contained 2,930 families drawn from a nationally representative sampling frame and 1,872 "SEO" families drawn from a low-income

sampling frame. Each year from 1968-1996, the PSID attempted to reinterview all persons living in families in the 1968 sample, as well as anyone born to or adopted by a previous PSID respondent. Our sample includes all households derived from the 1968 sample. We use sampling weights to adjust for the low-income over-sample and attrition. Our sample also includes the roughly 500 immigrant families added in 1997, but does not include the Latino sample added in 1990 and dropped after 1995. Also in 1997, the PSID stopped following roughly one-quarter of the original sample and began conducting interviews every other year. The survey has asked about food expenditure since its inception and in 2005 began asking about clothing, recreation and entertainment, and vacation expenditure.

To facilitate comparisons with the CE, we restrict the PSID sample to households with a head between 30 and 55 at the time of the interview, with no change in family composition between interview years, with real food expenditure in both years of at least 500 dollars, and to interview years between 1983 and 2012.

NIPA Consumption. We define C_t^{NIPA} as NIPA consumption of non-durable and non-housing services per person 16 years or older. We measure C_t^{NIPA} as follows. First, we obtain total nominal NIPA consumption of nondurable goods and non-housing services as the sum of NIPA table 2.4.5U lines 70 and 148 less line 151. Next, we construct a price index for this series as a Fisher aggregate of lines 70, 161, 168, 186, 205, 228, 246, 275, 284, 292, 301, 309, and 321 using NIPA tables 2.4.4U and 2.4.5U. We define C_t^{NIPA} as nominal NIPA consumption divided by the price deflator and by the number of persons aged 16 years or older.

ATUS. We use data from the 2003-2012 waves of the American Time Use Survey (ATUS). Individuals in the ATUS sample are drawn from the exiting sample of the CPS. For our estimates of the time endowment of the unemployed, we restrict the sample to respondents between the ages of 18 and 65 and with completed time diaries.

IRS Public Use Files. The IRS Public Use Files contain samples of anonymized U.S. federal income tax returns. The Public Use Files exist for 1960, 1962, 1964, and 1966-2008, containing

approximately 140,000 records per year. The files report detailed income tax information, including non-wage income such as dividends and capital gains, and information on deductions and credits.

UI Administrative Data. NIPA Table 2.6 (line 21) reports the dollar value of all benefits, by month, including extended benefit and emergency compensation tiers, based on unpublished data from the Employment Training Administration (ETA). The data begin in 1959.[2] Data on the number of claimants in all tiers come from the ETA for 1986-2013.[3] Prior to 1986, we collect data on the number of claimants in regular and extended benefits tiers from the statistical Appendix to the Economic Report of the President. Each year, the ERP lists the number of claimants in regular and extended benefit tiers, by month, for the previous two years. We digitize these data, seasonally-adjust the regular claims and benefits using X-11, and then add the unadjusted data for extended benefits tiers to form a single monthly time-series of recipients and benefits. Finally, we adjust the recipients series by the ratio of benefits payments for all tiers from the NIPA to benefits payments in regular and extended benefits tiers in the ERP to arrive at a series for the number of claimants in all tiers beginning in 1959.

Medicaid Administrative Data. NIPA Table 2.6 (line 20) reports the dollar value of Medicaid spending, by month.

Food Stamps/SNAP Administrative Data. For 1980-2012, we use monthly data on benefits disbursements from the Quality Control files maintained by Mathematica. The Quality Control files provide microdata on a representative sample of SNAP recipients used to assess program fraud and contain weights that aggregate up to the administrative total of recipients and benefits each month.[4] Prior to 1980, we use the annual dollar value as reported in NIPA Table 3.12 (line 21) and linearly interpolate over the year.

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[2]See http://www.bea.gov/national/pdf/mp5.pdf for a description of the source data for the NIPA estimates of UI, SNAP, TANF, and AFDC/TANF.

[3]http://ows.doleta.gov/unemploy/docs/persons.xls.

[4]See http://hostm142.mathematica-mpr.com/fns/2011/tech%20doc%202011.pdf for further description of the Quality Control data.

AFDC/TANF. NIPA Table 3.12 (line 35) reports the dollar value of AFDC/TANF spending, by year. We convert the annual total to monthly values by assuming that the within-year time path of spending equals the within-year distribution of caseloads. We obtain monthly caseloads for 1960-2011 from the U.S. Department of Health and Human Services Office of Family Assistance.[5]

Potential Duration of UI Eligibility. We set ω_t^u, the probability that an unemployed remains eligible, such that the expected potential duration of eligibility equals the national maximum, adjusted for the fact that not every unemployed individual has the maximal potential duration. Whittaker and Isaacs (2013) report the national maximum potential duration of UI receipt since the program's inception. Potential duration in most states depends on the worker's earnings history. For 1959-2013, data from the ETA give a mean potential duration of regular state benefits of 24 weeks, while the national maximum counts a potential duration of regular state benefits of 26 weeks. Additionally, benefits extensions under extended benefits or federal emergency programs may depend on a state's unemployment rate, such that not every state has a maximum potential duration equal to the national maximum. Unpublished data provided via email by Chad Stone and William Chen of the Center on Budget and Policy Priorities show that since 2008 the average state has had a maximum potential duration of 0.81 of the national maximum. Combining these two elements, we conservatively set the average potential duration to 0.8 of the national maximum potential duration.

Seasonal Adjustment. We seasonally adjust labor market variables from the basic monthly CPS files at a monthly frequency. Let y_t^n denote a not seasonally adjusted variable and y_t^s the seasonally adjusted variable. Our algorithm estimates 12 seasonal factors $\{\alpha_m\}$ from the ARMA(1,1) specification:

$$\ln y_{m,t}^n = \alpha_m + \rho \ln y_{m-1,t-1}^n + \theta e_{m-1,t-1} + e_{m,t}.$$

We then define $x_{m,t} = \ln y_{m,t}^n + (\bar{\alpha}_m - \alpha_m)$, where $\bar{\alpha}_m = \frac{1}{12} \sum_m \alpha_m$ is the average seasonal factor. For the hours per worker series, we also include in the ARMA model categorical variables for

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[5]http://archive.acf.hhs.gov/programs/ofa/data-reports/caseload/caseload_current.htm.

each of Good Friday, Easter Monday, Labor Day, Columbus Day, or Veteran's Day occurring during the CPS reference week and subtract the fitted values from $x_{m,t}^n$. We next apply the multistep moving average filter described in Findley, Monsell, Bell, Otto, and Chen (1998, Appendix A) and used in X-11 and exponentiate the resulting series to obtain $y_{m,t}^s$.

B.2 Heterogeneity Details

Benefits Estimation. For each group j, we measure $B_{k,jt}$ as:

$$B_{k,jt} = \left(\frac{\text{(survey dollars tied to unemployment status)}_{k,jt}}{\text{(total survey dollars)}_{k,t}} \right) \left(\frac{\text{(total administrative dollars)}_{k,t}}{\text{(number of unemployed)}_{jt}} \right), \quad (A.64)$$

where the first parenthesis is the share of program spending belonging in $B_{k,jt}$, called $B_{k,jt}^{\text{share}}$. Proceeding analogously to equations (14)-(15), we derive for group j:

$$\Delta y_{i,k,jt} = \beta_{k,jt}^0 + \beta_{k,jt} \Delta D_{i,jt}^u + \Delta \beta_{k,jt} D_{i,jt-1}^u + \Delta \epsilon_{i,k,jt}. \quad (A.65)$$

The share of program spending belonging in $B_{k,jt}$ is:

$$B_{k,jt}^{\text{share}} = \beta_{k,jt} \frac{\sum_{i \in j} \omega_{i,jt} D_{i,jt}^u}{\sum_i \omega_{i,jt} y_{k,i,jt}} = U_{jt}^{\text{share}} \beta_{k,jt} \left[\frac{\sum_i \omega_{i,jt} D_{i,jt}^u}{\sum_i \omega_{i,jt} y_{k,i,jt}} \right], \quad (A.66)$$

where $U_{jt}^{\text{share}} = \left[\frac{\sum_{i \in j} \omega_{i,jt} D_{i,jt}^u}{\sum_i \omega_{i,jt} D_{i,jt}^u} \right]$ is the share of unemployed belonging in category j. Substituting equation (A.66) into equation (A.65) gives:

$$\Delta y_{i,k,jt} = \beta_{k,jt}^0 + \left(\frac{B_{k,jt}^{\text{share}}}{U_{jt}^{\text{share}}} \right) \Delta \tilde{D}_{i,jt} + \Delta \beta_{k,jt} D_{i,t-1}^u + \Delta \epsilon_{i,k,jt}, \quad (A.67)$$

where we have used the definition $\Delta \tilde{D}_{i,jt} = \Delta D_{i,jt}^u \sum_i \omega_{i,jt} y_{k,i,jt} / \sum_i \omega_{i,jt} D_{i,jt}^u$.

Defining $B_{k,j}^0 \equiv \frac{B_{k,jt}^{\text{share}}}{U_{jt}^{\text{share}}}$ and constraining $B_{k,j}^0$ to be constant over time, we obtain our estimating equation:

$$\Delta y_{i,k,jt} = \beta_{k,jt}^0 + B_{k,j}^0 \tilde{D}_{i,t} + \Delta \beta_{k,jt} D_{i,t-1}^u + \Delta \epsilon_{i,k,jt}. \quad (A.68)$$

Equation (A.68) mirrors equation (17) for the aggregate, but with separate intercepts and slope coefficients by group. We estimate $B_{k,j}^0$ from equation (A.68) and finally construct $B_{k,jt}$ using

the following expression:

$$B_{k,jt} = B_{k,jt}^{\text{share}} \left(\frac{\text{total administrative dollars in category } k \text{ in period } t}{\text{number of unemployed in group } j \text{ in period } t} \right) \qquad (A.69)$$

$$= B_{k,j}^0 U_{jt}^{\text{share}} \left(\frac{\text{total administrative dollars in category } k \text{ in period } t}{\text{number of unemployed in group } j \text{ in period } t} \right)$$

$$= B_{k,j}^0 \left(\frac{\text{total administrative dollars in category } k \text{ in period } t}{\text{total number unemployed in period } t} \right) = B_{k,j}^0 \left(\frac{B_{k,t}}{B_{k,t}^{\text{share}}} \right).$$

Note that for any groups j and i, $B_{k,jt}/B_{k,it} = B_{k,j}^0/B_{k,i}^0$ and that $B_{k,t}^{\text{share}} = \sum_j B_{k,jt}^{\text{share}}$.

Benefits Expiration, Eligibility, and Take Up Disutility. We construct group-specific employment, unemployment, job-finding, separation, and UI eligibility using the CPS basic monthly microdata and the same procedures described in the main text for the aggregate. Using common $\omega_{jt}^e = \omega_t^e$ and $\omega_{jt}^u = \omega_t^u$, we construct the share of eligible unemployed by group ω_{jt} and the bracketed term in the expression for b_{jt} specific to each group. Lacking data on UI receipt by group, however, we cannot separately estimate the elasticity parameter α_j and take-up rates ζ_j. Instead, we impose $\alpha_j = \alpha$ and $\zeta_{jt} = \zeta_t$ for all groups j. Finally, we define $\phi_{jt} = \omega_{jt}\zeta_{jt}$ and $\tilde{B}_{jt} = B_{u,jt}/\phi_{jt}$.

Consumption Estimation. Denote by π_{jt}^e the fraction of the population 16 years or older who are employed and belong in group j, by π_{jt}^u the fraction of the population 16 years or older who are unemployed and belong in group j, by π_t^n the fraction of the population 16 years or older who are out of the labor force but of working age (16-64), and by π_t^r the fraction of the population 16 years or older who are older than 65 years old. With heterogeneity, the identity (24) expands to:

$$\sum_j \pi_{jt}^e C_{jt}^e + \sum_j \pi_{jt}^u C_{jt}^u + \pi_t^n C_t^n + \pi_t^r C_t^r = C_t^{\text{NIPA}}. \qquad (A.70)$$

We denote by $\gamma_{jt}^u = C_{jt}^u/C_{jt}^e$ the relative consumption of the unemployed in group j, by $\gamma_{jt}^e = C_{jt}^e/C_{it}^e$ the consumption of an employed in category j relative to an employed in category i, by $\gamma_t^n = C_t^n/C_t^e$ the consumption of someone out of the labor force but younger than 65 relative to the average employed, and by $\gamma_t^r = C_t^r/C_t^e$ the consumption of someone older than 65 relative to the average employed. Substituting these definitions into equation (A.70), we can

solve for consumption in some category i:

$$C_{it}^e = \frac{C_t^{\text{NIPA}}}{\sum_j \left(\pi_{jt}^e + \pi_{jt}^u \gamma_{jt}^u + \frac{\pi_{jt}^e}{\pi_t^e} \pi_t^n \gamma_t^n + \frac{\pi_{jt}^e}{\pi_t^e} \pi_t^r \gamma_t^r \right) \gamma_{jt}^e}. \tag{A.71}$$

To estimate γ_{jt}^u we follow the methodology described in Section 4.2, separately for each group j. To estimate γ_{jt}^e, we run the regression:

$$\ln C_{i,jt}^e = \left(\tilde{\gamma}_{jt}^e - 1 \right) \mathbb{I}\{i \in j\} + \phi \mathbf{X}_{i,jt} + \epsilon_{i,jt}, \tag{A.72}$$

where $\mathbf{X}_{jt}$ now excludes the controls that proxy for permanent income (financial asset variables and housing variables) but include the taste shock controls. Finally, we set $\gamma_{jt}^e = \exp\left(\tilde{\gamma}_{jt}^e - 1 \right)$.

As with the aggregate case, we apply the consumption ratios to equation (A.71) in two steps. First, the calibration of the preference parameters in Section 6 requires data on the mean level of C_{jt}^e (denoted by C_j^e) and the mean level of C_{jt}^u (denoted by C_j^u). For these, we impose constancy of the consumption ratios $\gamma_{jt}^e = \gamma_j^e$, $\gamma_{jt}^u = \gamma_j^u$, $\gamma_t^n = \gamma^n$, and $\gamma_t^r = \gamma^r$ in equation (25) and obtain a time series for C_{jt}^e for some reference group j. Recursively and applying the ratios γ_j^e and γ_j^u we obtain time series for all C_{jt}^e and C_{jt}^u. We then define $C_j^e = (1/T) \sum_t C_{jt}^e$ and $C_j^u = (1/T) \sum_t C_{jt}^u$.

Second, the time series of ξ_{jt} requires time series of C_{jt}^e and C_{jt}^u. We jointly impose the adding-up constraint for total consumption in equation (A.70) and four risk-sharing conditions for each j in equation (7) to solve for the time series of C_{jt}^e and C_{jt}^u.

Marginal Products. Defining $E_{jt} \equiv e_{jt} l_j$ as the total employment in group j, we start with the production function:

$$Y_t = A_t K_t^\nu \left(\left(\sum_j \nu_j \left(N_{jt} E_{jt} \right)^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} \right)^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}} \right)^{1-\nu}, \tag{A.73}$$

with $\sum_j \nu_j = 1$. The pre-tax marginal products are given by:

$$p_{jt} = \frac{\partial Y_t}{\partial E_{jt}} = p_t^e \left(\frac{E_t}{E_{jt}} \right) \left(\frac{\nu_j \left(E_{jt} N_{jt} \right)^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}}}{\sum_i \nu_i \left(E_{it} N_{it} \right)^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}}} \right). \tag{A.74}$$

where $p_t = (1 - \nu)Y_t/E_t$ denotes the aggregate marginal product of employment. Note that the employment-weighted average of the marginal products equals the aggregate marginal product:

$$\sum_j \left(\frac{E_{jt}}{E_t} \right) p_{jt} = p_t. \tag{A.75}$$

We calibrate the ν_j's such that the ratio of the pre-tax marginal products equals the ratio of pre-tax labor earnings:

$$\frac{p_{jt}}{p_{it}} = \frac{w_{jt}N_{jt}}{w_{it}N_{it}}. \tag{A.76}$$

Letting ν_{jt} denote values that make this equation hold exactly in period t, and using $\sum_j \nu_{jt} = 1$, we obtain:

$$\nu_{jt} = \frac{w_{jt} \left(E_{jt}N_{jt} \right)^{1/\tilde{\sigma}}}{\sum_i w_{it} \left(E_{it}N_{it} \right)^{1/\tilde{\sigma}}}. \tag{A.77}$$

We calibrate the ν_j as the in sample averages of ν_{jt}:

$$\nu_j = \frac{1}{T} \sum_{t=1}^{T} \frac{w_{jt} \left(E_{jt}N_{jt} \right)^{1/\tilde{\sigma}}}{\sum_i w_{it} \left(E_{it}N_{it} \right)^{1/\tilde{\sigma}}}. \tag{A.78}$$

Mean wages w_{jt} by educational group are estimated from the CPS March Supplement as the ratio of total labor earnings to total hours worked for those respondents who reported at least 20 hours of work per week throughout the year. In the calculations above, we assume an elasticity of substitution $\tilde{\sigma} = 5$. Our results do not change significantly when we use a value of $\tilde{\sigma} = 2$.

C   Incomplete Asset Markets

In this appendix we discuss a model with incomplete asset markets. We show that the opportunity cost implied by this model is equal to the sum of the z from equation (10) in the main text and an additional component which we denote by z^A. Therefore, the z we measure in the data does not constitute a sufficient statistic for unemployment fluctuations in the incomplete markets model. Measuring z^A requires finding an empirical counterpart for the value function, a task that goes beyond the scope of this paper. However, when we calibrate a version of the incomplete markets model in this appendix, we find that the term z^A is generally small and does not offset the procyclicality of z.

We consider the problem of an individual who cannot share risks perfectly with other members of the household, but instead accumulates assets a_t to self insure against idiosyncratic employment shocks. Assets earn a net rate of return equal to r_t. Individuals face the borrowing constraint $a_t \geq \bar{a}_t$. The transitions to and out of employment in the form of job finding rates f_t and separations s_t are treated as exogenous aggregate states from the point of view of workers. Let $\pi(\mathbf{Z}_{t+1}|\mathbf{Z}_t)$ denote the probability that the aggregate state transits from $\mathbf{Z}_t$ to $\mathbf{Z}_{t+1}$.

Denote by a_{t+1}^e the choice of assets for period $t+1$ conditional on being employed in period t. The value function of an employed who starts with assets a_t is:

$$W^e(a_t, \mathbf{Z}_t) = U(C_t^e, N_t) + \beta \sum_{\mathbf{Z}_{t+1}} \pi(\mathbf{Z}_{t+1}|\mathbf{Z}_t) \left((1-s_t)W^e(a_{t+1}^e, \mathbf{Z}_{t+1}) + s_t W^u(a_{t+1}^e, \mathbf{Z}_{t+1})\right),$$

(A.79)

subject to the budget constraint $\left(1 + \tau_t^C\right) C_t^e + a_{t+1}^e + \Pi_t = (1 - \tau_t^w) w_t N_t + (1+r_t)a_t$. Similarly, denote by a_{t+1}^u the choice of assets for period $t+1$ conditional on being unemployed in period t. The value function of an unemployed who starts with assets a_t is:

$$W^u(a_t, \mathbf{Z}_t) = U(C_t^u, 0) + \beta \sum_{\mathbf{Z}_{t+1}} \pi(\mathbf{Z}_{t+1}|\mathbf{Z}_t) \left(f_t W^e(a_{t+1}^u, \mathbf{Z}_{t+1}) + (1-f_t)W^u(a_{t+1}^u, \mathbf{Z}_{t+1})\right),$$

(A.80)

subject to the budget constraint $\left(1 + \tau_t^C\right) C_t^u + a_{t+1}^u + \Pi_t = b_t + (1+r_t)a_t$. For simplicity, we abstract from UI eligibility and the take-up decision and simply lump these margins into b_t. Finally, we note that Π_t now also includes transfers that do not depend on employment status.

An individual entering period t with assets a_t receives a surplus from moving from unemployment to employment equal to $J_t^h = W^e(a_t, \mathbf{Z}_t) - W^u(a_t, \mathbf{Z}_t)$. We define $J_{t+1}^h = W^e(a_{t+1}^e, \mathbf{Z}_{t+1}) - W^u(a_{t+1}^e, \mathbf{Z}_{t+1})$. Evaluating both terms of J_{t+1}^h at a_{t+1}^e restricts the $t+1$ surplus to only that part associated with entering $t+1$ in the employed state. Substituting (A.79) and (A.80) into J_t^h, we obtain:

$$\frac{J_t^h}{\lambda_t^e} = \left(\frac{1 - \tau_t^w}{1 + \tau_t^C}\right) w_t N_t - z_t - z_t^A + (1 - s_t - f_t)\mathbb{E}_t \left(\frac{\beta \lambda_{t+1}^e}{\lambda_t^e}\right) \frac{J_{t+1}^h}{\lambda_{t+1}^e}, \qquad (A.81)$$

where z_t is defined again as in equation (10) in the main text and z_t^A denotes a component of the opportunity cost related to the differential asset accumulation between the employed and the

unemployed.[6] We divide by the marginal utility of the employed λ_t^e because the wage negotiated during bargaining is paid in the state of the world in which the individual accepts the offer.

In our analysis with full risk sharing, we attributed the entirety of the 21 percent decline in consumption upon unemployment to non-separabilities between consumption and hours. Here we make the opposite extreme assumption that the consumption decline results only from market incompleteness. We, therefore, restrict our analysis to the separable preferences SEP presented in the main text.

We choose the level of income not related to labor force status, Π, such that, in the space of assets considered below, consumption drops on average by 21 percent upon unemployment. The exogenous state vector $\mathbf{Z}_t$ includes the employment rate e_t, the separation rate s_t, the wage w_t, hours per employed worker N_t, and benefits per unemployed b_t. We define an aggregate "good state" in which the shocks e_t, w_t, and N_t are one standard deviation above their trend and s_t and b_t are one standard deviation below their trend. We define an aggregate "bad state" symmetrically.[7]

Figure A.1 plots the opportunity cost of employment in the models with perfect risk sharing (left panel) and with self insurance (right panel) for different starting assets and aggregate states. Several results are worth highlighting. First, the opportunity cost is in general lower in the model with self insurance. With imperfect risk sharing, workers save more to insure against idiosyncratic shocks. Lower consumption for a given level of assets means that the marginal utility of consumption for both the unemployed and the employed is higher relative to the model with perfect risk sharing. As initial assets increase, the probability of hitting the borrowing constraint becomes smaller, and the opportunity cost in the model with self insurance

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[6]We have $z_t^A = -\frac{\beta}{\lambda_t^e}\mathbb{E}_t\left[f_t\left(W^e(a_{t+1}^e, \mathbf{Z}_{t+1}) - W^e(a_{t+1}^u, \mathbf{Z}_{t+1})\right) + (1 - f_t)\left(W^u(a_{t+1}^e, \mathbf{Z}_{t+1}) - W^u(a_{t+1}^u, \mathbf{Z}_{t+1})\right)\right] + a_{t+1}^e - a_{t+1}^u$. Moving from unemployment to employment (holding constant initial assets at a_t) causes a "budgetary loss" equal to $a_{t+1}^e - a_{t+1}^u$ due to the fact that employed accumulate more assets. There is an offsetting gain as the individual starts $t+1$ with higher assets. Because all of the surplus associated with a higher probability of having a job in $t+1$ is included into J_{t+1}^h, the value function gains from entering $t+1$ with assets a_{t+1}^e instead of a_{t+1}^u are evaluated as if the individual obtains employment in period $t+1$ with probability f_t.

[7]To match a 21 percent decline in consumption, we set $\Pi = -1.13$. We calibrate $\chi = 0.74$ to match an opportunity cost in the risk-sharing model equal to 0.75 in the space of assets shown in Figure A.1. We set the borrowing constraint to $\bar{a} = -0.5$, which corresponds to a fraction 23% of the total non-capital income of the employed $wN - \Pi$. The rest of the parameters are $w = 1$, $N = 1$, $b = 0.058$, $\beta = 0.98$, $r = 0.01$, $\epsilon = 0.7$, $s = 0.045$, $f = 0.704$, and $\tau^w = \tau^C = 0$. Finally, we discretize the state in three values and assume that the transition matrix is given by $\pi\left(\mathbf{Z}_{t+1} = \mathbf{Z}_j|\mathbf{Z}_t = \mathbf{Z}_i\right) = 0.98$ for $j = i$ and $\pi\left(\mathbf{Z}_{t+1} = \mathbf{Z}_j|\mathbf{Z}_t = \mathbf{Z}_i\right) = 0.01$ for $j \neq i$.
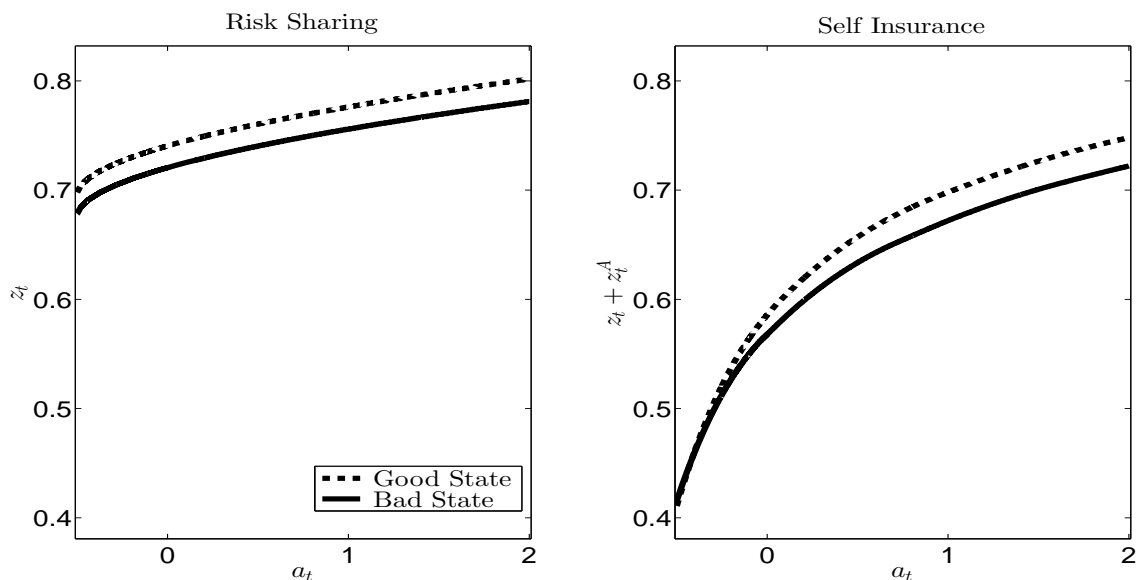
Figure A.1: Opportunity Cost Under Alternative Risk Sharing Arrangements

Notes: The left panel plots the opportunity cost of employment for the model with perfect risk sharing in a good state (dashed line) and in a bad state (solid line). The right panel plots the opportunity cost of employment for the model with self insurance.

approaches the level of the opportunity cost in the model with perfect risk sharing.

Second, in both models the opportunity cost increases in the level of assets. The increase is much sharper for workers close to the borrowing constraint in the model with self insurance. These workers have a very high marginal utility of consumption, making them more desperate to work. The positive relationship between assets and the opportunity cost implies procyclical movements in the opportunity cost in both models if in recessions the average wealth of the unemployed declines.

Third, in both models the opportunity cost is in general procyclical for a given level of assets, as evidenced by the upward shift of the dashed line relative to the solid line. Just as with complete markets, with incomplete markets the marginal utility of consumption falls relative to the value of non-working time in the good state. For very low levels of initial assets, however, the opportunity cost becomes less cyclical.[8]

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[8]Nakajima (2012) develops a model with incomplete markets, leisure, and a benefit replacement rate of 64% that generates high volatility in unemployment. He argues that changes in borrowing constraints do not matter much for the performance of search and matching models as workers save and self-insure sufficiently to overcome these constraints. The opportunity cost generated by his model may be less cyclical than what we estimate because benefits constitute two-thirds of his opportunity cost.

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